### CHAPTER 2

# ENTREPRENEURS UNDER AMBIGUITY

# **A Prospect Theory Perspective**

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### **ABSTRACT**

This chapter focuses on entrepreneurial decision making under ambiguity. Our main contribution is to insist on the importance of ambiguity in entrepreneurship, as many entrepreneurial decisions are highly strategic, unique, and mostly taken in situations with limited and imprecise information. Uncertainty, quintessential in entrepreneurship, has inspired early theoretical works and psychological analysis of decision making in the entrepreneurship literature. Past contributions have mainly focused on risk and risk preferences as determinants of entry in entrepreneurship, but the message of this literature is hardly conclusive. In this chapter, we provide a state of the art overview on a recently emerging literature on ambiguity attitudes and entrepreneurship research. Many of such recent works are based on Expected Utility Theory and thus carry the same limitations of this theory. Our chapter emphasizes the advantage of experimental economics and the potential of Prospect Theory applications. We focus specifically on the weighting function

aspect, and theorize on how it can contribute to further our understanding of entrepreneurial behavior. Prospect Theory represents a sophisticated tool that may disentangle subtle behavioral differences between entrepreneurial profiles and other decision makers.

Entrepreneurial decisions, being at the forefront of innovation and economic development, involved with the creation or discovery processes of lucrative opportunities, are continuously subject to ambiguity (Knight, 1921; Shane & Venkataraman, 2000). Success stories in entrepreneurship are associated with many funding anecdotes. For example, in 1999, graduate students Brin and Page, founders of Google.com, offered to sell for \$1 million the search engine they had developed to George Bell, CEO of Excite, who then rejected the offer. Within that year, Google.com obtained \$25 million of funding, and today the company is worth \$416.8 billion of market capitalization.

Embracing the unknown is intrinsic in entrepreneurship (Hayek, 1948), although not all undertakings are similar in their potential achievements and riskiness. Many entrepreneurs perish after ill-conceived market strategies, persistently trying alternative ventures, and mostly accepting losses and significant trade-offs in terms of income (Hamilton, 2000). Moreover, numerous startups flourish in Western economies, but many fail soon after inception: 50% fail within two years and 67% within four years (Geroski, 1995), a phenomenon known as "excess entry". Although the existing entrepreneurship research has advanced several explanations, as over-optimism or escalation of commitment, these observations are still considered open questions and there is no unifying theory for the entrepreneurial choice (Astebro, Herz, Nanda, &Weber, 2014). Such empirically documented observations also contradict most economic models based on Expected Utility Theory, as these entrepreneurial choices do not appear to be wealth maximizing. Moreover, under conditions of uncertainty and ambiguity, with no probabilities available, the classical economic model is silent on behavioral predictions, let alone interpretations of observed behavior. Research in entrepreneurship, typically multidisciplinary, has developed into a stream of studies following a psychological tradition, pointing to specific traits and possible judgment biases that may influence entrepreneurial decision making. Thus far, decades of studies have investigated the degree of optimism of entrepreneurs, overconfidence, risk preferences, ambiguity perceptions, and so forth. However, these past efforts have failed to produce strong evidence of a typical "entrepreneurial profile," and whether entrepreneurs are more risk tolerant, as posited in most occupational choice theories, is yet unsettled. Additional contributions that have measured ambiguity tolerance of entrepreneurs and other decision makers, relying on psychometric scales (Dollinger, 1983; Schere, 1982) have also provided inconsistent or opposing results.

Although uncertainty is quintessential in entrepreneurship and entrepreneurial decision making, the ambiguity that characterizes most strategic and unique decisions is often neglected in management and entrepreneurial studies. With the development of theories that allow investigating ambiguity quantitatively, there is a renewed interest in addressing ambiguity attitudes in entrepreneurship research. Ambiguity refers to situations where probabilistic information is imprecise and cannot be inferred from existing statistical data (Knight, 1921). Situations of ambiguity in the business domain could refer to strategic decisions, to creation of new ventures, to choices of career paths, to resource allocation processes or other financial commitments. In situations of ambiguity estimating probabilities of success is challenging, as these decisions are often unique with no available history, are strongly dependent on contingencies and offer little opportunities for learning.

In decision contexts under ambiguity, Ellsberg (1961) in his seminal contribution predicts *ambiguity aversion*, which describes individuals' choice to refrain from options with unknown probabilities, and prefer risky options instead, with known probabilities. In fact, ambiguity in life or business generates a sort of "discomfort" that may hinder action. When considering economic decisions with no clear course of action to be taken, people may mull over expectations of possible scenarios, postpone decisions, or delay choices, in order to collect more relevant information, which may not always be available.

As an example, one could consider a developer evaluating two alternative platforms for launching his new application. Both platforms yield the same outcome in terms of profit in case of success. However, for the first platform, the developer knows that for every hundred new applications proposed, fifty succeed and fifty fail. The rates of success for the other platform are unknown. What is a sensible decision in this case? What would determine the developer's choice? Or, would he simply decide on the basis of perceptions about his own abilities to develop a successful application? The developer may be enticed to invest time and other resources in finalizing his application anyway, and decide at a later stage on the platform. He would thus be vulnerable to the ambiguity of such decision. Maybe he overestimates the unlikely event of success, given the considerable number of other developers in his domain; or maybe he underestimates the highly likely event of failure, like many other developers did before him. How can we account for such behavioral patterns?

Advances in Prospect Theory have made possible to better understand behavior under both risk and ambiguity. Very recently, emerging experimental designs in entrepreneurship are employing Prospect Theory to explain observed entrepreneurial behavior (Hsu, Wiklund, & Cotton, 2015;

Shyti & Paraschiv, 2015). In this chapter, we offer a state of the art perspective of ambiguity theories and recent developments of experimental economics in the context of entrepreneurship. We draw insights on how we can better understand micro foundations of entrepreneurship based on behavioral theories of choice, as Prospect Theory.

The rest of the chapter is organized as follows. First, the chapter offers a brief review of the literature on uncertainty and ambiguity. Second, it focuses on the weighting function aspect of Prospect Theory. It continues by summarizing the existent literature on entrepreneurial decision making under risk and ambiguity. The chapter concludes by emphasizing the potential contributions of Prospect Theory and experimental economics in future developments in entrepreneurship research.

# DECONSTRUCTING UNCERTAINTY: A DECISION MAKING PERSPECTIVE

The importance of uncertainty in economic activity was understood since Cantillon in 1755, when he first pointed out that the entrepreneur sustains certain costs of production while facing uncertain profits that manifest over time. Other scholars recognize that entrepreneurs are rewarded for bearing uncertainty rather than risk, which is in turn insurable (Knight, 1921; Say, 1836). In the same vein, subsequent formal theories posit that the more risk tolerant individuals self-select in entrepreneurship and hire the risk averse individuals as employees (Kihlstrom & Laffont, 1979).

However, tackling empirically uncertainty bumps into obvious operationalization difficulties. Knight (1921) offered a clear distinction between risk and uncertainty. He defined risk as a situation in which possible events (and associated consequences) are known and there is a probability distribution over the possible events. Knight (1921) defined uncertainty as a situation in which possible events are known, but there is no probability distribution available to the decision maker. This is also known as *Knightian Uncertainty*, different from Radical Uncertainty of Keynes (1921) and Shackle (1968) (see Figure 2.1). Following Ellsberg (1961), the definition of ambiguity focuses on the absence of probabilities, thus locating ambiguity as an in-between situation of two extremes: the *risky* option with known probabilities on one pole, and *uncertainty*, the option with no available probabilities on the other.

Situations of risk are characterized by known probabilities, and are mostly encountered in games of chance, in lotteries, or in psychological experiments. In real-life circumstances precise probabilities are rarely available. Surprisingly, past research in management has focused mainly on risk and risk perceptions of managers, investors, and other decision makers. The same trend has characterized also entrepreneurship research, in which

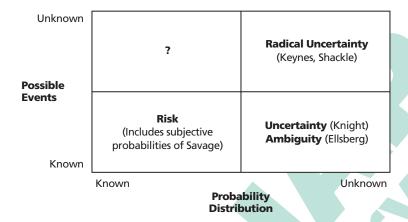


Figure 2.1 Types of uncertainty. Source: Adapted from Wakker (2010)

uncertainty has been treated qualitatively (McKelvie, Haynie, & Gustavsson, 2011; McMullen & Shepherd, 2006), and often as a synonym of risk (Alvarez & Barney, 2005; Shane, 2000).

## Ellsberg's Paradox

Savage (1954) is among the most prominent works in decision theory that formally represented uncertainty in individual decision making. In Subjective Expected Utility Theory, Savage (1954) introduces the notion of *subjective probability distribution* of decision makers. More precisely, uncertainty is seen as subjective and the absence of objective probabilities is considered not to affect decision making. In fact, in absence of objective probabilities, decision makers assign their subjective probabilities to any world event, and are able to evaluate economic options through the classical Expected Utility Theory.

Ellsberg (1961) was the first to formally show that for a decision maker a risky option is not the same as an ambiguous option, which is characterized by the absence of probabilistic information. Ellsberg's work proved that ambiguity affects decision making, as decision makers are not indifferent to it. Ellsberg (1961) shows that decision makers systematically avoid ambiguous options, and prefer risky options instead. This choice pattern causes decisions to deviate from Expected Utility Theory predictions. Ellsberg called this phenomenon *ambiguity aversion*, which is also known as Ellsberg's Paradox.

In Ellsberg's famous Two-Color example, a decision maker faces two urns, each containing 100 balls. The Known Urn contains 50 red balls and 50 black balls. The Unknown Urn contains 100 balls, either red or black,

but in unknown proportion. The decision maker is asked to choose one urn (Known or Unknown) and one color (red or black). Once he makes his choice, a ball is drawn randomly from the selected urn. If the ball is the color chosen by the decision maker, he wins a prize; otherwise, he gets nothing. The decision maker is indifferent regarding the betting color (red or black) within each urn. However, the decision maker is not indifferent between the two urns. The phenomenon that Ellsberg (1961) describes is that the decision maker, unable to assign objective probabilities to either red or black in the Unknown Urn, refrains from that option altogether, exhibiting ambiguity aversion.

Following Ellsberg (1961)'s lead, two parallel streams of research developed in decision science. The first stream of research, with an axiomatic orientation, mostly produced works that model ambiguity aversion (Gilboa & Schmeidler, 1989; Schmeidler, 1989). In almost all these decision models ambiguity aversion was assumed to be an invariant feature of the decision maker's preferences. The second stream of research, with a descriptive aim, derived most of its results from experimental studies, with the objective to understand decision making under ambiguity. Experimental investigations have confirmed ambiguity aversion, but have also shown that individuals do not always avoid options with unknown probabilities as in Curley and Yates (1985), Einhorn and Hogarth (1985), Fox and Tversky (1995), and Wu and Gonzalez (1999), to mention some. Studies that have addressed decision under ambiguity from a descriptive perspective include Budescu, Khun, Kramer, and Johnson (2002), Curley and Yates (1989), González-Vallejo, Bonazzi, and Shapiro (1996), and Kuhn and Budescu (1996). Other approaches have tested the impact of ambiguity on different models of behavior, most prominent works based on prospect theory including Fox and Tversky (1998), Gonzalez and Wu (1999), Hogarth and Einhorn (1990), Kilka and Weber (2001), Tversky and Fox (1995), Tversky and Wakker (1995), Wakker (2004), and Wakker (2010). Yet, another set of studies, including Cohen, Jaffray, and Said (1987), Curley and Yates (1985), and Heath and Tversky (1991), have investigated the consequences of ambiguity on choice.

# **Operationalizing Ambiguity**

Previous studies, mostly belonging to the psychological stream of research, have treated the concept of ambiguity with probability intervals (Budescu et al., 2002; Curley & Yates, 1985; González-Vallejo et al., 1996; Smithson, 1999), a business compatible way to address ambiguity. Let's reflect on the example of the developer who considers launching the application on a commercial platform. The entrepreneur is contemplating whether he could reach one million downloads in the first semester. Imagine he

seeks the opinion of a tech expert and receives an estimate of 15% rate of success. However, for the same one million downloads in the first semester, the platform owner provides an estimate of 45% rates of success. Thus, the developer is facing an interval [15%, 45%] within which may lie the rate of success of achieving one million downloads in the first semester. These estimates naturally generate a probability interval.

Another example of ambiguity represented by intervals comes from Amazon.com report in the Second Quarter 2014 Guidance stating the following:

net sales are expected to be between 18.1 billion and 19.8 billion, or to grow between 15% and 26% compared with second quarter 2013 " and "operating income (loss) is expected to be between \$455 million and \$55 million, compared to \$79 million in second quarter 2013.

In this example ambiguity refers to outcomes (e.g., operating income, percentage of growth, etc.), instead of probabilities. Thus, such representations of ambiguity as intervals of information may apply to other business contexts, in which experts and consultants provide their best estimates about success rates of a project, the probability of occurrence of an event, the probability of an event occurring within a time horizon, the earnings forecast of a company, and so forth.

A natural propensity of decision makers in business is to reduce uncertainty by gathering more information or seeking a third party opinion. Restricting the boundaries or noisy information allows for the application of more precise decision rules. In conditions of uncertainty, there is naturally a greater need to apply heuristics, for lack of better rules, but uncertainty also increases the potential of decision biases and errors (Kahneman, Slovic, & Tversky, 1982; Kahneman & Tversky, 1973). It emerges from dialogues with business practitioners that when they have the option to demand more information, they rarely ask probabilistic information, for instance, the probability to achieve a certain rate of return, or the probability to accomplish a project on time (Kahneman, 2011). However, this does not mean that business practitioners ignore the uncertainty of their environments and its impact on decision making.

### PROSPECT THEORY FOR RISK AND AMBIGUITY

One of the most prominent modern decision theories developed to analyze uncertainty quantitatively is Prospect Theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992; Wakker, 2010). This theory is the result of the devotion and work of Amos Tversky and Daniel Kahneman, the latter

famous for earning the Nobel Prize in Economics in 2002 and being regarded as the father of *behavioral economics*. Prospect Theory was proposed as a reaction to the failure of Expected Utility Theory to adequately explain behavior under risk. Therefore, it accounts for a large series of biases in individual decision making.

### **The Weighting Function**

An important idea in Prospect Theory is the distortion of probabilities in the decision making process through a probability weighting function, w(.). Consider a decision maker evaluating a risky prospect that yields an outcome X with probability p and nothing otherwise. The value of such prospect under Prospect Theory will depend on the importance of outcome X for the decision maker and also on his subjective perception of the probability p. Prospect Theory models the fact that individuals do not behave according to the objective or given probability p, but interpret subjectively this probability by attributing more or less weight to it, through a decision weight, w(p). In general, Prospect Theory is concerned with prospects, which evaluation is considered independent of decisions maker's total wealth. Formally, the value a decision maker attributes to the risky prospect depends on his utility function, U(.), and on the probability weighting function for risk, wr(.), and equals  $U(X) \times wr(p)$ . Under Prospect Theory, risk attitudes of decision makers are partially reflected in the shape of the utility function U(.), and partially reflected in the shape of the weighting function for risk wr(.).

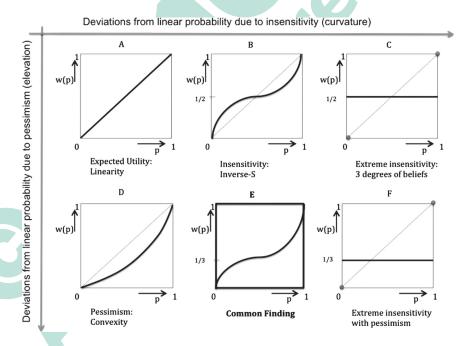
Ambiguous prospects are evaluated under Prospect Theory in a very similar way, meaning through a decision maker's utility function and a weighting function. When dealing with ambiguous prospects, a common approach in experimental research has consisted in assuming a different weighting function for ambiguity, wa(.). Ambiguity attitudes of decision makers reflect the differences in behavior under risk and under ambiguity. Assuming that the utility function is the same under both risk and ambiguity, ambiguity attitudes of decision makers are thus captured by the differences between weighting functions for risk, wr(.), and ambiguity, wa(.).

### **Experimental Evidence on Weighting Functions for Risk**

Typical empirical evidence shows that decision makers exhibit risk seeking for gains with low probabilities and risk aversion for gains with high probabilities. The general interpretation of these findings in terms of the non-linear treatment of probabilities assumed by Prospect Theory is that

individuals overestimate small probabilities and underestimate large probabilities. Graphically the shape of w(.) can be represented through an inverted S-shape probability weighting function as shown in Figure 2.2E. The idea behind the inverted S-shape probability weighting function is that the decision maker is willing to pay more to pass from an impossible event (0%) to a possible one (10%) (the concave part near 0 in Figure 2.2E); as well, the decision maker is willing to pay considerable amounts to shift from a highly likely event (90%) to a sure one (100%) (the convex part near 1 in Figure 2.2E). However, decision maker's willingness to pay to improve the probability of an event from 40% to 50% is much lower.

The probability weighting function captures two distinct psychological phenomena in decision making: insensitivity and pessimism. Likelihood *insensitivity* refers to the flattened perception for changes in intermediate probabilities, which are perceived close to the 50% probability. This phenomenon reflects decision makers' inability to sufficiently discriminate between probability levels. Kahneman and Tversky (1979) attribute this phenomenon to cognitive psychological causes, and assume that these distortions take place during an "editing" phase, before the decision maker attaches any value to the prospects under consideration. According to Wakker



**Figure 2.2** Weighting functions under prospect theory. *Source:* Adapted from Abdellaoui et al. (2011)

(2010), likelihood insensitivity is irrational and can be corrected through incentives and learning. *Pessimism* refers to the tendency to underestimate the chances of success and is reflected graphically by the distance of the curve from the *x*-axis. For gains or positive outcomes, a weighting function closer to the *x*-axis is associated with more pessimism, corresponding to an underweighting of probabilities. On the contrary, a weighting function further from the *x*-axis is associated with optimism and corresponds to inflation or overweighting of probabilities.

Empirical studies on Prospect Theory generally rely on functional forms to provide an overall picture on decision maker's risk attitude. The literature offers a variety of functional forms for the weighting function (Abdellaoui, l'Haridon, & Zank, 2010; Goldstein & Einhorn, 1987; Kahneman & Tversky, 1979; Prelec, 1998), that provide the advantage of summarizing information on decision maker's probability weighting through aggregated indexes. When information is summarized using one single index, it is difficult to establish a direct relation between the index itself and a clear psychological phenomenon. Therefore, recent literature recommends functional forms based on two indexes, related to *elevation* and *curvature* (Abdellaoui, Baillon, Placido, & Wakker, 2011). Such forms have the advantage to disentangle the two psychological phenomena discussed before: the pessimism/ optimism of the decision maker captured by an elevation parameter and the sensitivity to changes in probabilities captured by a curvature parameter. One of the most commonly used functional form is Prelec's (1998) two-parameter specification, given in the Equation 2.1:

$$ww(p) = \left(\exp\left(-\left(-\ln(p)\right)\right)a\right)b \tag{2.1}$$

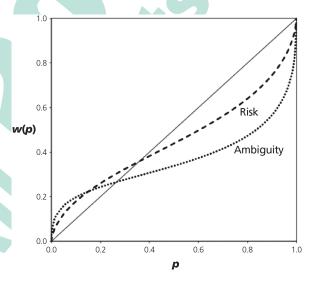
where a is an insensitivity index (curvature) and b is a pessimism index (elevation). When both indexes are 1, the weighting function does not present any distortion and corresponds to the  $45^{\circ}$  line. Parameter a values reflect insensitivity to changes in likelihood, with lower values corresponding to more insensitivity. Parameter b values reflect pessimism, with higher (lower) b values corresponding to higher degrees of pessimism (optimism).

While the inverse S-shaped form in Figure 2.2E with pessimism and insensitivity is the most common finding in experimental research, experiments also report substantial variation of individual behavior. Figure 2.2 depicts several possible patterns of behavior. Figure 2.2A corresponds to the Expected Utility model with a decision maker treating probabilities linearly, with no pessimism and no insensitivity. The other figures correspond to different combinations of insensitivity and/or pessimism: Figure 2.2B reflects pessimism; Figure 2.2C reflects extreme insensitivity, corresponding to a decision maker who has the same behavior regardless of the probabilities involved; and Figure 2.2F reflects extreme insensitivity with pessimism.

# **Experimental Evidence on Weighting Functions for Ambiguity**

Although in the last decades the topic of decision making under ambiguity has received a lot of attention in decision theory, empirical evidence is still limited. Abdellaoui et al. (2011) is one of the first studies that tests experimentally Prospect Theory under ambiguity. The authors show that the inverted S-shape of the weighting function is preserved under ambiguity, but distortions are even more pronounced for extreme likelihoods. The study reports ambiguity aversion for mid-range and high likelihoods. Figure 2.3 represents the weighting function for risk, wr(.), clearly different from the 45° line, as well as the weighting function for ambiguity, wa(.). The line of ambiguity shows that low likelihoods are even more overweighted compared to the case of risk, and larger likelihoods are even more underweighted. Abdellaoui et al. (2011) also report that ambiguity attitudes differ according to decision contexts, for typical lottery experiments and natural sources of uncertainty (e.g., investing in the French stock market).

Taken together, these findings show that the idea of a weighting function is powerful and some key lessons emerge from the decision making literature that need to be considered in entrepreneurship research. First, decision maker's risk and ambiguity attitudes are distinct (for a comprehensive literature review, see Camerer & Weber, 1992) and should not be treated as surrogates of each other. As individual attitudes toward risk are conceptually distinct from attitudes toward ambiguity, it's important for future



**Figure 2.3** Weighting functions for risk and ambiguity.

research in entrepreneurship to analyze these attitudes in complement to one another. Second, risk and ambiguity attitudes are not consistent with expected utility maximization. Because the utility-maximizing argument of Expected Utility Theory is questionable when introducing ambiguity, the experimental designs used to study ambiguity attitudes of entrepreneurs need to address the issue of probability weighting. Third, emerging evidence shows that risk and ambiguity attitudes are not invariant traits of behavior, but depend on the decision context. Experimental studies, while a valuable tool to understand ambiguity attitudes of entrepreneurs, should investigate behavior using frameworks directly related to entrepreneurial decisions. Finally, in order to assess decision maker's behavioral responses to risk and ambiguity it is not sufficient to use only one probability point, but several questions scanning the entire probability range. This will allow to account for entrepreneurs' behavior not only for intermediate probabilities, but also for low probabilities and high probabilities.

# ENTREPRENEURIAL DECISION MAKING UNDER RISK AND AMBIGUITY

The premises of decision making under ambiguity set the ground for further exploitations of Prospect Theory in entrepreneurship and managerial studies aimed at uncovering micro foundations of behavior in business environments.

# **Entrepreneurs Under Risk**

An overwhelming body of literature in entrepreneurship research has considered individual risk preferences as an important determinant of entry to entrepreneurship and has employed a variety of approaches to empirically measure risk aversion. Occupational choice theories have supported the *risk tolerance* assumption of entrepreneurs (Douglas & Shepherd, 2000; Kanbur, 1979; Kihlstrom & Laffont, 1979). However, Brockhaus's (1980) efforts to measure individual risk propensities using psychometric scales in univariate investigations reported no differences between entrepreneurs and non-entrepreneurs, a result later supported by Gartner (1988). The hypothesis of *risk tolerant* entrepreneurs found some empirical validation in longitudinal surveys (Caliendo, Fossen, & Kriticos, 2009; Cramer, Hartog, Jonker, & Van Praag, 2002), which showed that the willingness to take risks increases the probability that an individual enters entrepreneurship in the future.

Risk attitudes as an entrepreneurial trait were also investigated through economic experiments with binary lotteries. Works of Elston and Audretsch

(2011) and Elston, Harrison, and Rutstrom (2005) have reported that entrepreneurs are either risk neutral or slightly risk averse, but again, no specific differences were detected between entrepreneurs and other decision makers.

Clearly, such multitude of methods and variety of results do not enable to draw definitive conclusions. Typical critiques on the psychometric scales to measure risk propensities emphasize that risk aversion or risk seeking could be conflated with over-optimism and other individual traits might influence respondents' answer patterns. Referring to the survey method, Caliendo et al. (2009) point to the difficulties and challenges of measuring risk aversion in the field. The longitudinal approach was further criticized as risk attitudes may not be stable over time. Also the experimental economics approach to investigating differences in risk attitudes of entrepreneurs and non-entrepreneurs (Elston & Audretsch, 2011; Elston et al., 2005) is not free of limitations. This approach was mostly criticized for using abstract lotteries with small-stakes, which focus on one probability point and assume Expected Utility Theory.

Such multi-disciplinary evidence in entrepreneurship research does not allow to draw clear conclusions on an association of specific risk attitudes with entrepreneurial profiles (Parker, 2009). Moreover, these considerations call for more attention on the fundamental point raised by Dohmen, Falk, Huffman, Sunde, Schupp, and Wagner (2011) on the stability or risk attitudes across contexts and over time.

### **Entrepreneurs Under Ambiguity**

The general inconclusiveness of risk attitudes to define entrepreneurial profiles has not discouraged research efforts to also investigate entrepreneurs' attitudes toward ambiguity.

Entrepreneurs start a new venture with a vague knowledge of their likelihood of success. In many occasions, entrepreneurs lack experience and face conflicting or insufficient statistical evidence, conditions that make it difficult to define precise predictions about the success of their business. Challenges related to estimates of likelihood success for a new venture are also related to the aggregation that the notion of success incorporates. Success can be related to surviving on a 10-year horizon, to obtaining initial funding, to achieving a 20% market share within a given time span. Most entrepreneurs believe that their fund raising campaigns will succeed and the Venture Capitalist decisions will be favorable for their startup. Thus, addressing how entrepreneurs perceive their chances of success in the field is challenging, as investors' feedback, market's response, or customers' appreciation may manifest in some future undefined time horizon. The initial investigations of the topic of ambiguity appeared in the '80s, with a predominantly psychological approach. Recently, a few experimental economics contributions focus on specific aspects of entrepreneurial decision making under ambiguity. Yet, how ambiguity influences entrepreneurs' decisions is still an under-investigated topic in entrepreneurship research, despite its essential role in understanding entrepreneurial behavior.

### Psychological Experiments in Entrepreneurship

Ambiguity is not new in psychological research. During the late forties, Frenkel- Brunswik (1949) was the first to conceive and develop a psychometric scale aimed at assessing individual perceptions to ambiguity that she named *intolerance to ambiguity*. Budner (1962) modified the original *intolerance to ambiguity* scale and its interpretation, defining an ambiguous situation as one "which cannot be adequately structured or categorized by the individual because of the lack of sufficient cues or situations characterized by novelty, complexity, or insolubility" (p. 30), with "threatening" or "avoidance" reactions to ambiguity manifesting through cognitive, emotional, and behavioral aspects. Also Mac Donald Jr (1970) further revisited the *intolerance to ambiguity* psychometric scale switching to *ambiguity tolerance*, to convey a framing in which ambiguity was desirable.

Schere (1982), adopting a trait-approach to entrepreneurship research, investigated ambiguity perception of entrepreneurs and managers relying on applications of the ambiguity intolerance scale. Schere (1982) reported entrepreneurs to exhibit higher ambiguity tolerance compared to managers, results that confirmed his hypothesis of entrepreneurs facing highly uncertain situations, characterized as turbulent, chaotic, complex or conflicting.

Later on, Dollinger (1983) failed to replicate the same findings, and attributed such inconsistency to poor sampling. However, more recent contributions (Tajeddini & Mueller, 2009; Teoh & Foo, 1997) examine a variety of entrepreneurial personal characteristics, including the *intolerance to ambiguity* scale, with mainly inconclusive results. Generally, such psychometric scales have been subject of criticism for being unreliable, for providing piecemeal results on the psychology of ambiguity perceptions, weak links to decision making, and for lacking strong contributions to theory (McLain, 2009).

### Economic Experiments in Entrepreneurship

Only very recently, a few empirical papers have addressed ambiguity attitudes of entrepreneurs, tapping in from the vast experimental literature in decision making. Hardenbol (2012) interviewed entrepreneurs, managers, and students, and reported no difference among their choice behavior. Hardenbol (2012) estimates ambiguity and risk attitudes using binary choices between lottery options, with gains up to \$40. Bengtsson, Sanandaji, and Johannesson (2012) investigate ambiguity and risk for entrepreneurs and non-entrepreneurs based on a survey with 11,743 individuals from

the Swedish Twin Registry. Risk attitudes are inferred through individuals' choices between a fixed and a variable salary with probability 50%. Ambiguity attitudes are determined through a single question, based on three-color Ellsberg's example (Ellsberg, 1961), in which respondents have to choose between a risky lottery and an ambiguous lottery. Bengtsson et al. (2012) findings are consistent with less risk and ambiguity aversion for entrepreneurs compared with non-entrepreneurs. Koudstaal, Sloof, and Van Praag (2014) run a large scale lab-in-the-field experiment. They gather data from a survey with 910 entrepreneurs, 397 managers and 981 employees in Holland. This design employs tasks with multiple choice lists for both risk and ambiguity using a 50% probability. In the case of risk the decision maker chooses between a risky option and a certain option. In the case of ambiguity, the task involves a risky option and an ambiguous option. The results of Koudstaal et al. (2014) reveal that entrepreneurs and managers are equally ambiguity averse, and slightly more ambiguity averse compared to employees. However, they report that these differences disappear when controlling for typical demographics as age, education and income among others.

Probably, the most prominent study to date on entrepreneurial behavior under uncertainty is provided by Holm, Opper, and Nee (2013) and compares 700 entrepreneurs and 200 non-entrepreneurs based in China. These authors study risk and ambiguity attitudes using several binary choices between monetary lotteries. Their main tool to determine ambiguity attitudes is a decision task in the form of a choice list with one option offering a sure outcome, and the other option offering an outcome with ambiguous probabilities between 25% and 75%, whose natural center is 50%. In another task, they use an uncertainty option (no probability information provided) instead of the ambiguous one. They observe that compared to the control group, entrepreneurs were more willing to accept situations of uncertainty involving competition and trust. As per risk and ambiguity attitudes, general results of Holm et al. (2013) report no differences between entrepreneurs and the control group.

# **Entrepreneurs Under Prospect Theory**

The above-mentioned experimental designs (Bengtsson et al., 2012; Hardenbol, 2012; Holm et al., 2013) do not sufficiently account for the existing evidence on Prospect Theory. They usually employ a Holt and Laury (2002) method based on Expected Utility, thus providing biased estimates for risk and ambiguity attitudes. Extensive empirical evidence shows that Expected Utility is a fallible guide to understand decision making, as individuals consistently violate its predictions. Also, decision tasks are often based on Ellsberg-type lottery questions with relatively low stakes

(Bengtsson et al., 2012; Hardenbol, 2012). Such tools, based on lotteries, are of questionable validity when used to predict behavior for business decisions as ambiguity attitudes are not constant across domains (Abdellaoui et al., 2011; Dohmen et al., 2011). Moreover, ambiguity attitudes are usually investigated for the 50% likelihood level, thus neglecting the richness of behavior that occurs at extreme likelihoods.

To the best of our knowledge, only one recent study by Shyti and Paraschiv (2014) attempts to investigate entrepreneurial behavior under ambiguity using Prospect Theory. Shyti and Paraschiv (2014) show that both entrepreneurs and wage earners behave according to Prospect Theory (Kahneman & Tversky, 1979). This is perhaps the first study that emphasizes violations to Expected Utility Theory predictions in entrepreneurial decisions making. The paper reports the results of an online experiment aimed at comparing attitudes towards risk and towards ambiguity in occupational choice decisions for a group of entrepreneurs and a group of nonentrepreneurs. Respondents are presented with a series of potential entrepreneurial projects, differing in the degrees of risk and ambiguity, and are asked to state their wage equivalent for each project. Based on the reported wage equivalents, the authors estimate weighting function for risk, wr, and for ambiguity, wa. They provide evidence of inverted S-shaped weighting functions for both entrepreneurs and non-entrepreneurs, consistent with overestimation of low likelihoods and underestimation of high likelihoods under risk and under ambiguity. The use of Prospect Theory to analyze occupational choice decisions allows providing a very precise picture concerning behavioral differences between entrepreneurs and non-entrepreneurs. The behavior of the two groups is different, with entrepreneurs exhibiting a higher level of optimism compared to non-entrepreneurs. Overall, entrepreneurs are more risk seeking and more ambiguity averse in evaluating entrepreneurial projects. Shyti and Paraschiv (2014) report prevailing pessimism for entrepreneurs under ambiguity, consistent with entrepreneurs being more sensitive to the precision of information about the chances of success of an entrepreneurial project.

#### **IMPORTANCE OF PROSPECT THEORY**

Bridging experimental work in decision theory and entrepreneurship research is a challenging and not vacuous task. Besides the importance of ambiguity and uncertainty in entrepreneurship, yet very little is known regarding entrepreneurial behavior under ambiguity. Although scholars have suggested that biases influence entrepreneurial decisions, and more so under uncertainty (Busenitz & Barney, 1997; Schade & Koellinger, 2007), the specific direction of influence and the micro mechanisms of this association

(i.e., whether cognitive or motivational) remain unclear. Hence, it becomes relevant to understand entrepreneurial decision making under ambiguity, and then address the role of relevant biases coupled with entrepreneurial behavioral responses. Modern behavioral theories, specifically Prospect Theory, may account for some of the unresolved empirical puzzles in entrepreneurship. For instance, Prospect Theory could offer alternative explanations for the observed over-entry in markets. Overweighting of small probabilities may contribute to risk seeking and excess entry, which may be consistent with escalation of commitment or over-investing, and may lead to high rates of entrepreneurial failure. On the other extreme of the probability range, underweighting of high probabilities may relate to underinvesting in profitable prospects. Hence, Prospect Theory may account for the richness of behavior that we observe empirically in the business world. However, many challenges remain, and to examine business decisions one needs to take into consideration several likelihood levels, including the extremes (e.g., very low and very high probabilities). Additionally, such theory is versatile and enables to deal with behavioral perceptions of ambiguity, the total absence of probabilities, and different degrees of ambiguity. Moreover, the enhanced descriptive power of Prospect Theory may allow detecting also subtle differences in decision making of entrepreneurs and non-entrepreneurs through carefully designed experiments.

#### CONCLUSION

This chapter contributes to the ongoing debate on the role of uncertainty in entrepreneurial decision making by focusing on ambiguity. Our main contribution is to insist on the importance of ambiguity in entrepreneurship, as many entrepreneurial decisions are taken under conditions of imprecise information. We argue that an entrepreneur is rarely in a situation with known probabilities (risk) or in a situation in which he knows nothing at all (complete ignorance or radical uncertainty), but in an intermediate state in which he has a vague idea about the chances of success. Our second message is that, in order to adequately investigate ambiguity, entrepreneurship scholars should build on modern decision theories. A considerable body of economic research in entrepreneurship is still based on Expected Utility Theory, which is shown to be a fallible guide in understanding behavior due to its normative nature. Prospect Theory can provide a better framework to study ambiguity.

Our chapter can also be seen as a state of the art of experimental economics applications to investigations of ambiguity in entrepreneurial decision making. The scant empirical evidence that has emerged so far in the entrepreneurship field confirms the challenges and constraints in addressing ambiguity in decision making. The "late" interest towards ambiguity attitudes in entrepreneurship is partly due to the complexity of models that analyze uncertainty quantitatively and partly to the difficulties to adapt experimental designs to the context of entrepreneurial decisions. The increasing number of empirical studies during the last years attests to a vivid debate on entrepreneurship research on attitudes toward ambiguity, although the message of these emerging studies is yet scattered, and does not allow to fully grasp differences in behavior or typical patterns in entrepreneurial decision making. Nonetheless, the current approach to investigating the topic of ambiguity in entrepreneurial decision making stresses the importance of economic experiments as a promising method that could further our understanding of behavior profiles of entrepreneurs and non-entrepreneurs.

However, so far many open questions remain. What do we know about ambiguity attitudes of entrepreneurs in different business contexts? Do entrepreneurs accommodate ambiguity in their decision processes and under which conditions? Do entrepreneurs differ from non-entrepreneurs and what are the factors that explain these differences? These questions call for further investigations of ambiguity based on experimental economics and behavioral decision making theories that provide the advantage to focus on particular circumstances and to assess the role of specific factors. A promising direction for future research in entrepreneurship is to explore the role of entrepreneurial experience. Two recent studies point to the importance of this factor. First, a study by Hsu et al. (2015) puts the accent on the contradicting predictions of self-efficacy theory of individuals that restart a business after experiencing failure in a previous business venture. Assuming Prospect Theory, Hsu et al. (2015) find experimental support for the observed entrepreneurial re-entry. A second study, by Shyti and Paraschiv (2015), suggests that startup experience might reduce ambiguity aversion of entrepreneurs. Thus, serial entrepreneurs (that have more than two startup experiences) are shown to be more optimistic and less ambiguity averse than novice entrepreneurs (that have only started a business once). These findings call for further research on factors that moderate the relation between entrepreneurial experience and ambiguity attitudes, as expertise, human capital, accumulated wealth, and so forth.

In investigating behavioral differences between entrepreneurs and nonentrepreneurs, another important topic for future research might also be to refine the relation between the retained definition of an entrepreneur (Gartner, 1988) and observed ambiguity attitudes. Potential contributions could assess ambiguity attitudes of specific types of entrepreneur, such as innovators (Schumpeter, 1934), arbitrageurs (Kirzner, 1973), or simply self-employed individuals. Future research could also focus on the degree of ambiguity, as the existence of more or less ambiguity may bear some relevance on observed behavior of entrepreneurs. Higher degrees of ambiguity might yield more prudence, which could be more pronounced for entrepreneurs than for non-entrepreneurs. Thus, higher ambiguity might be associated with higher ambiguity aversion.

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